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State Feedback Robust Tracking Controller Based on Preview-H_∞ Control Approach

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Abstrak

Di dalam makalah ini diusulkan sebuah metoda baru merancang pengendali tracking menggunakan pendekatan kendali preview-H_∞(H tak terhingga). Nama kendali preview-H_∞ pada makalah ini memiliki arti kombinasi antara teori kendali preview dan teori kendali H_∞ . Dengan mengasumikan bahwa jalur lintasan objek radar dapat diprediksi untuk beberapa waktu pencuplikan ke depan, maka informasi tentang sinyal referensi masa depan ini dapat digunakan untuk membuat pengendali trakcing yang memiliki kinerja yang lebih baik. Di lain pihak, teori kendali H_∞ digunakan untuk membuat sistem kendali lebih kokoh (robust) terhadap gangguan internal maupun eksternal seperti gaya tiupan angin. Pada makalah ini tata kendali umpan balik keadaan telah diformulasikan berdasarkan prinsip maksimum dan teori defferential game. Selain itu, hubungan antara fungsi pinalti dan H_∞-norm juga telah dianalisa.

Kata kunci: umpan balik keadaan, kokoh, tracking, pengendali, preview, kendali kokoh H_∞ (tak terhingga), radar, target, lintasan, jalur.

Abstract

This paper proposes a novel design method of a state feedback robust tracking controller based on preview-H_∞ (infinity) control approach. The name of preview-H_∞ control in this paper implies the combination between preview control theory and H_∞ control theory. By assuming that the radar target future travel path can be predicted for certain number of sampling period, the future information of reference signal can be utilized to construct a tracking controller with better tracking performance. On the other hand, the H_∞ control theory has been used to make the control system more robust against internal and external disturbances including wind force. The formulation of state feedback preview-H_∞ control law has been carried out by the use of the maximum principle and differential game theory. Moreover, the relation between the cost function and H_∞-norm criterion has been analyzed.

Key words: state feedback, robust, tracking, controller, preview, H_∞ control, radar, target, travel, path.

1.Introduction

A tracking radar should follows the position of one or more objects in space, ignoring the content of the space not occupied by the target(s) being tracked. In Single Target Track (STT) the radar follows a single object and ignores all others. In Multi Target Track (MTT) the radar continuously monitors the position of several targets, with each target sampled many times per second. Effectively,one radar performs the function of as many tracking radars as there are targets being tracked. Multi-target tracking requires that the antenna's beam position be changeable essentially instantaneously, and this is normally possible only with electronically scanned antennas. For true tracking to take place, the target(s) must be sampled at the Nyquist rate for the track servo bandwidth and target maneuvering bandwidths [1].

By the use of the so called $\alpha - \beta$ filter whose parameters are tuned using the Kalman filter algorithm the radar target travel path can be predicted [2]. When the radar target travel path undergoes totally unpredictable high-G maneuvers it is often desired to use multiple tracking gates [3].

By assuming that the radar target future travel path can be estimated for certain number of sampling period during operation, the future

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information of the reference signal can be utilized to construct a tracking control system with better tracking performance. The control method which incorporates the future information is well known as preview control. So far the preview control has been analyzed in the frame work of Linear Quadratic Regulator (LQR) or Linear Quadratic Gaussian (LQG) control theory, and the preview control system has been designed to optimize a cost function of Linear Quadratic form which contains certain state variables, reference signals, and control input needed. It has been known that, compared with feedback LQR/LQG, the preview control makes further reduction of the minimum value achievable by the cost function. Also the preview control results in a zero-phase control system in which the phase of the state variable being controlled is the same as that of the reference signal for relatively low frequency domain [4]. In spite of these advantages, the preview control has the lack in that it does not take into consideration the effects of un-certain disturbance.

The disturbances may be present in two types, i.e., internal disturbance due to un-modeled dynamics or parametric variation, and external disturbance such as friction forces, wind forces, and so on. Using H_∞ control theory, these disturbances can be taken into consideration in the design stage of the control system. The specifications such as stability robustness against internal disturbance and low sensitivity against external disturbance are reduced to a criterion in terms of H_∞ norm by means of appropriate weighting functions [7], [8]. However, the feedback H_∞ control uses only information corresponding to the present state value, and the limitations such phase-delay will probably be encountered.

This paper presents a novel method for designing a state feedback robust tracking controller based on preview- H_∞ control approach which is a combination of preview control and H_∞ control. The analytical formulation of the state feedback preview-H_∞ control law has been derived in the continuous time domain.

2. Optimal Preview Control

In this section, for the sake of simplicity, a linear dynamic system as shown in state space equation (1) is considered. Here, x_1, x_2 are the state variables of the plant to be controlled, and x_e is a manipulated state variable which is obtained by integrating the error signal between x_2 and a reference signal r. In this paper, the integral action is incorporated to ensure zero steady-state error. u is the control input. Evaluation equation is expressed in equation (4) where ($z_{1,}\ z_{21,} z_{22}$) are evaluation signals and (η_1, η_2) are weighting coefficients which should be adjusted during the design of the controller. During the formulation of the optimal preview control law disturbance w_1 due to uncertainty of either the plant or the environment which is not a priory known is not used.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_e \end{bmatrix} + \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \quad (1)$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} A & 0 \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_e \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (2)$$

Or simply,

$$\dot{x} = Ax(t) + B_1 u(t) + B_r r(t), \quad (3)$$

where: $x(t_0) = x_0$.

$$\begin{bmatrix} z_1 \\ z_{21} \\ z_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_e \end{bmatrix} + \begin{bmatrix} \xi \\ 0 \\ 0 \end{bmatrix} u \quad (4)$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ C_2 & C_{2_e} \end{bmatrix} \begin{bmatrix} x \\ x_e \end{bmatrix} + \begin{bmatrix} \xi \\ 0 \end{bmatrix} u \quad (5)$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ C_2 \end{bmatrix} \begin{bmatrix} x \\ x_e \end{bmatrix} + \begin{bmatrix} \xi \\ 0 \end{bmatrix} u \quad (6)$$

$$z(t) = Cx(t) + Du(t). \quad (7)$$

Here, the following cost function to be minimized is considered,

$$J = \frac{1}{2} z_2'(t_f) S z_2(t_f) + \frac{1}{2}\int_{t_0}^{t_f} \left[z_2'(t) Q\, z_2(t) + u'(t) R u(t) \right] dt, \quad (8)$$

where: $S, Q, R = \xi'\xi$ are positive definite matrices. Let the Hamiltonian be defined as follows:

$$H(x,u,\lambda,t)=\frac{1}{2}z_2'(t)Q\,z_2(t)+\frac{1}{2}u'(t)Ru(t)$$
$$+\lambda'(t)\left[Ax(t)+B_1u(t)+B_r r(t)\right], \quad (9)$$

where $\lambda(t)$ is the Lagrange multiplier. Using the maximum principle, in order to minimize the cost function, the following must hold [5],

$$\frac{\partial H}{\partial u}=0 \quad (10)$$

Which yields,

$$u=-R^{-1}B_1'\lambda(t), \quad (11)$$

And

$$\frac{\partial H}{\partial x}=-\dot{\lambda}=C_2'QC_2x(t)+A'\lambda(t) \quad (12)$$

With the terminal condition,

$$\lambda(t_f)=\frac{\partial\left\{\frac{1}{2}z_2'(t_f)Sz_2(t_f)\right\}}{\partial x(t_f)}=C_2'SC_2x(t_f)\,. \quad (13)$$

To determine the control law, it is assumed that,

$$\lambda(t)=P(t)x(t)+q(t)\,. \quad (14)$$

From equations (3), (11), (12), and (14), the following differential equations are obtained,

$$\dot{P}(t)+P(t)A+A'P(t)$$
$$-P(t)B_1R^{-1}B_1'P(t0+C_2'QC_2=0 \quad (15)$$

$$\dot{q}(t)+\left[A-B_1R^{-1}B_1'P(t)\right]'q(t)$$
$$+P(t)B_r r(t)=0 \quad (16)$$

where the boundary conditions are,

$$P(t_f)=C_2'SC_2=P_{t_f} \quad (17)$$

$$q(t_f)=0 \quad (18)$$

The control law is then given by,

$$u(t)=-R^{-1}B_1'\left[P(t)x(t)+q(t)\right]$$
$$=-R^{-1}B_1'P(t)x(t)-R^{-1}B_1'q(t)$$
$$=u_b+u_f \quad (19)$$

where $P(t)$ and $q(t)$ are the solution of equation (15) and equation (16), respectively. The total control input is composed by feedback control input u_b and feed-forward control input u_f. Therefore, the closed loop system can be illustrated as in figure 1. P is the dynamical system to be controlled, $K=-r^{-1}B_1'P(t)$, and F_f represents the reference signal processor calculating $q(t)$ according to equation (16).

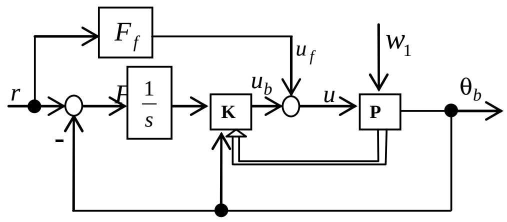

Figure 1. The Closed Loop System

3. Preview- H_∞ Control

3.1. Reference Signal a Priori Known for Total Working Time $[t_0, t_f]$

Taking into consideration the uncertain disturbance w_1 and setting $\xi=1$, the state-space equation (3) and the evaluating equation (7) can be expressed by the following equations:

$$\dot{x}=Ax(t)+B_1u(t)+B_r r(t)+B_w w_1(t) \quad (20)$$

where $x(t_0)=0$, and

$$z=\begin{bmatrix}z_1\\ z_2\end{bmatrix}=\begin{bmatrix}0\\ C_2\end{bmatrix}\begin{bmatrix}x\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u \quad (21)$$

Equation (20) can be expressed as follows

$$\dot{x}=Ax(t)+\begin{bmatrix}B_w & B_1\end{bmatrix}\begin{bmatrix}w_1\\ u\end{bmatrix}+B_r r(t) \quad (22)$$

$$\dot{x}=Ax(t)+Bv+B_r r \quad (23)$$

Suppose that the reference signal $r(t)$ is known during the total working time $[t_0, t_f]$, and let a cost function be defined by,

$$\bar{J}=\frac{1}{2}z_2'(t_f)\bar{S}z_2(t_f)+$$
$$\frac{1}{2}\int_{t_0}^{t_f}[z_2'z_2+\begin{bmatrix}w' & u'\end{bmatrix}\begin{bmatrix}-\gamma^2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}w_1\\ u\end{bmatrix}]dt \quad (24)$$
$$=\frac{1}{2}z_2'(t_f)\bar{S}z_2(t_f)+\frac{1}{2}\int_{t_0}^{t_f}\left[z_2'z_2+v'\bar{R}v\right]dt \quad (25)$$

where γ is a given positive value.

Analogy to the previous section, the cost function $\bar{J}_{[t_0,t_f]}$ is optimized by

$$w_1(t)=\gamma^{-2}B_w'\bar{P}(t)x(t)+\gamma^{-2}B_w'\bar{q}(t) \quad (26)$$

$$u(t)=-B_1'\bar{P}(t)x(t)-B_1'\bar{q}(t)\,, \quad (27)$$

where $\bar{P}(t), \bar{q}(t)$ are solutions of the following differential equations,

$$\dot{\bar{P}}(t) + \bar{P}(t)A + A'\bar{P}(t) - \bar{P}(t)\left[B_1B_1' - \gamma^{-2}B_wB_w'\right]\bar{P}(t) + C_2'C_2 = 0 \quad (28)$$

$$\dot{\bar{q}}(t) + \left[A - (B_1B_1' - \gamma^{-2}B_wB_w')\bar{P}(t)\right]'\bar{q}(t) + \bar{P}(t)B_r r(t) = 0 \quad (29)$$

The boundary conditions are,

$$\bar{P}(t_f) = C_2'\bar{S}C_2 > 0 \quad (30)$$

$$\bar{q}(t_f) = 0. \quad (31)$$

In order to analyze the relationship between the cost function and H_∞-norm criterion, the following mathematical definition is introduced [6].

$$1) \int_0^T (z'z - \gamma^2 w'w)dt \le -\varepsilon\|w\|^2_{2[0,T]} \Leftrightarrow \left\|\frac{z}{w}\right\|_{[0,T]} < \gamma$$

$$2) \int_0^\infty (z'z - \gamma^2 w'w)dt \le -\varepsilon\|w\|^2_{2[0,T]} \Leftrightarrow \left\|\frac{z}{w}\right\|_\infty < \gamma$$

where ε is a small positive parameter.

When the transfer function matrix from uncertain disturbance w_1 to the evaluation signal z is denoted by

$$\frac{z(s)}{w_1(s)} = T_{zw}(s) \quad (32)$$

the design criterion in terms of H_∞-norm can be expressed as follows

$$\|T_{zw}\|_\infty < \gamma. \quad (33)$$

According to the induced norm concept, this criterion is equivalent with the following criterion [6].

$$J_\infty = \int_0^\infty [z(t)'z(t) - \gamma^2 w_1(t)'w_1(t)]dt < 0. \quad (34)$$

On the order hand, assuming $t_f = t_0 + T$ and $t_0 = 0$, form the cost function in equation (25), the following cost function can be defined

$$J_{[T,F]} = x_T'Fx_T + \int_0^T [z(t)'z(t) - \gamma^2 w_1(t)'w_1(t)]dt, \quad (35)$$

where $F = \bar{P}(t_f)$.

Let define a Lyapunov function of the energy as:

$$V = x'\bar{P}x, \quad (36)$$

Then the following equation must hold,

$$\int_0^T [x'\dot{\bar{P}}x + \dot{x}'\bar{P}x + x'\bar{P}\dot{x}]dt - x'\bar{P}x\Big|_0^T = 0 \quad (37)$$

Substituting equation (23) into equation (37) the following equation is obtained,

$$\int_0^T [x'(\dot{\bar{P}} + A'\bar{P} + \bar{P}A)x + (x'\bar{P}B_1u + u'B_1'\bar{P}x) + (x'\bar{P}B_w w_1 + w_1'B_w'\bar{P}x)]dt - x'\bar{P}x\Big|_0^T + \int_0^t (x'\bar{P}B_r r + r'B_r'\bar{P}x)\,dt = 0. \quad (38)$$

Since adding equation (38) into the cost function of equation (34) does not change the value, the cost function can be expanded as follows,

$$J_{[T,F]} = x_T'Fx_T - x'\bar{P}x\Big|_0^T + \int_0^T [x' \quad w_1' \quad u'] \begin{bmatrix} \alpha & \bar{P}B_w & \bar{P}B_1 \\ B_w'\bar{P} & -\gamma^2 & 0 \\ B_1'\bar{P} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ w_1 \\ u \end{bmatrix} dt + \int_0^T (x'\bar{P}B_r r + r'B_r'\bar{P}x)dt. \quad (39)$$

where $\alpha = (\dot{\bar{P}} + A'\bar{P} + \bar{P}A + C_2'C_2)$.

By substituting $\dot{\bar{P}}$ of equation (28) into equation (39) and recalling that $x(0) = 0$ yields

$$J_{[T,F]} = \int_0^T [u + B_1'\bar{P}x]'[u + B_1'\bar{P}x]dt - \gamma^2\int_0^T [w_1 - \gamma^{-2}B_w'\bar{P}x]'[w_1 - \gamma^2 B_w'\bar{P}x]dt + \int_0^T (x'\bar{P}B_r r + r'B_r'\bar{P}x)\,dt \quad (40)$$

If $r(t) = 0$, the third term in the right side of equation (40) become zero. In this case the saddle point is given by

$$u(t) = u^*(t) = -B_1'\bar{P}(t)x(t) \quad (41)$$

$$w(t) = w^*(t) = \gamma^{-2}B_w'\bar{P}(t)x(t) \quad (42)$$

where u^*, w^* are optimum solution for controlled input and disturbance, respectively. Here, w^* can be interpreted as the worst disturbance.

Therefore, the cost function above can be expressed by

$$J_{[T,F]} = \|u - u^*\|^2_{2,[0,T]} - \gamma^2\|w - w^*\|^2_{2,[0,T]} \quad (43)$$

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for any controlled input u and any disturbance w. For the optimum control input, equation (43) means that

$$\|T_{zw}\|_{[0,T]} < \gamma . \quad (44)$$

If T is assumed to be infinity, then $\dot{\bar{P}} = 0$, and the H_∞ norm criterion of equation (33) or equation (34) is satisfied.

When $r(t) \neq 0$, by substituting the optimum $w_1(t)$ in equation (26) and u in equation (27) into the cost function in equation (40), the following is obtained

$$J_{[T,F]opt} = \int_0^T [(q^{-1}(t)(B_1 B_1' - B_w B_w')\bar{q}(t)]dt + \int_0^T [x'\bar{P}B_r r + B_r'\bar{P}x]dt \quad (45)$$

According to the induced norm, the following relation are obtained

$$\|T_{zw}\|_{[0,T]} = J_{[T,F]opt} - x_T' F_{XT} \quad (46)$$

$$\|T_{zw}\|_{[\infty]} = J_{[\infty,F]opt} - x_\infty' F_{X\infty} \quad (47)$$

Therefore when the reference signal $r(t)$ is present, the H_∞-norm criterion of the transfer function from the disturbance to the evaluation signal is affected by it.

3.2 Reference Signal Previewed for a Certain Short Interval of Time $[t_0, t_0 + \tau]$

In this case, it is assumed that the reference signal $r(t)$ is known only during a certain interval of time τ which is much lesser than the total working time T which is assumed to be long enough. During the interval $[t_0, t_0 + \tau]$ the following cost function is considered

$$J_1 = \int_{t_0}^{t_0+\tau} [z(t)'z(t) - \gamma^2 w_1(t)'w_1(t)]dt . \quad (48)$$

Since for the rest interval of time $[t_0, t_0 + \tau]$ the reference signal is unknown, it is considered as uncertain disturbance along with $w_1(t)$. Assuming that T is infinity, during this interval the following cost function is considered

$$J_2 = \int_{t_0+\tau}^{\infty} [z(t)'z(t) - \gamma^2 w(t)'w(t)]dt . \quad (49)$$

Where

$$w = \begin{bmatrix} r(t) \\ w_1(t) \end{bmatrix} . \quad (50)$$

Therefore, total cost function to be optimized is

$$J_{tot} = J_1 + J_2 . \quad (51)$$

From linear Optimal control Theory it is well known that the optimum value of the cost function J_2 is

$$J_2 = x'(t_0 + \tau)\hat{P}x(t_0 + \tau) . \quad (52)$$

where $\hat{P}$ is the solution of the following Algebraic Riccati Equation

$$\hat{P}A + A'\hat{P} + C_2'C_2 - \hat{P}[B_1 B_1' - \gamma^{-2}(B_r B_r' + B_w B_w)]\hat{P} = 0 \quad (53)$$

From equations (49), (51), (52) the cost function J_{tot} can be expressed as:

$$J_{tot} = x'(t_0 + \tau)\hat{P}x(t_0 + \tau) + \int_{t_0}^{t_0+\tau} [z'(t)z(t) - \gamma^2 w_1'(t)w_1(t)]dt \quad (54)$$

Since the cost function of equations (54) and (35) are similar with the only difference in their terminal conditions, the control law which optimizes the cost function J_{tot} of equations (54) subject to the constraint of equation (20), is also given by equation (27) with $\hat{P}(t)$ calculated from equation (28) and $\bar{q}(t)$ calculated from equation (29).

More over, since the cost function of equation (54) and (35) are similar with the only difference in their terminal conditions, the analysis of H_∞-norm of the transfer function from the disturbance $w_1(t)$ to the evaluation signal $z(t)$ can be carried out in similar way as in the previous subsection

4. Conclusions and Discussions

The preview- H_∞ control law has been derived which has the feedback and feed forward parts. The control law is given by solving a Riccati equation and by calculating a differential equation of the signal used in the feed forward part. Concerning the prediction of future value of reference signal, two results have been obtained, i.e., reference signal previewed for total working time $[t_0, t_f]$, and reference signal previewed for a certain short interval of time $[t_0, t_0 + \tau]$.

From the analysis of H_∞-norm criterion it has been found that the reference signal has an effect on the value of the H_∞-norm of the transfer function from the disturbance to the evaluation signal.

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References

[1] Byron Edde, ”RADAR: Principles, Technology, Applications”, Prentice Hall Inc., New Jersey, 1993.
[2] T. R. Benedict and G. W. Bordner, ”Synthesis of an Optimal Set of Radar Tracking-While-Scan Smoothing Equations”, IRE Transactions, vol.AC-7, pp.27-32, July 1962.
[3] G. V. Trunk, ”Survey on Radar ADT”, Naval Research Laboratory Report 8698, June 1983.
[4] M. Hayase and K. Ichikawa, ”Optimal Servo Systems Utilizing Future Value of Desired Function”, Journal of the society of instrumentation and control engineers, No.1, Vol.5, pp.86-94, Japan, 1969.
[5] A.P. Sage and C.C. White III, ”Optimum System Control”, Prentice-Hall, New Jersey, 1977.
[6] M. Green and D.J.N. Limbeer, ”Linear Robust Control”, Prentice-Hall, New Jersey, 1995
[7] B.A. Francis, ”A Course in H_∞ Control theory”, Lecture Notes in Control and Information Sciences, Springer-Verlag, Heidelberg, 1987
[8] Doyle J.C., Glover K., Khargonekar P., and Francis B., ”State-Space Solutions to Standard H_2 and H_∞ Control Problems”, IEEE Trans. on Automatic Control, vol.34, No.8,1989.