Design of Radar Antenna Tracking Servo Using State Feedback Robust Tracking Controller Based on Preview-H_∞ Control Approach

Estiko Rijanto

Pusat Penelitian Tenaga Listrik dan Mekatronik (Puslit Telimek),
LEMBAGA ILMU PENGETAHUAN INDONESIA (LIPI),
Research Center for Electrical Power and Mechatronics,
INDONESIAN INSTITUTE OF SCIENCES,
Jl. Cisitu No.21/154D, Bandung 40135, Indonesia; email: estiko@hotmail.com.

Abstract

This paper presents an application of preview-H_∞ control method on designing a radar antenna tracking servo. Assuming that the radar target future travel path can be estimated for certain number of sampling period during operation, the future information of the reference signal can be utilized to construct a tracking control system having better tracking performance towards the reference signal as well as robustness against disturbance. In this paper, the preview-H_∞ control uses state feedback. The tracking performance towards the reference signal is analyzed both in frequency domain and in time domain. The frequency responses are compared with those of the control systems designed using either only H_∞ feedback control or conventional optimum preview control. To evaluate the time response, reference signals of type of sinusoidal and sigmoid function have been applied. From the numerical analysis results obtained in this study it has been verified that the control system with preview-H_∞ controller has better tracking performance than that which uses only H_∞ feedback controller, and also that it performs better disturbance attenuation characteristics than a control system designed using only optimum preview controller.

Keywords: radar, antenna, tracking, servo, state feedback, preview, H_∞, controller.

Abstrak

Makalah ini mengetengahkan sebuah aplikasi metoda kendali preview-H_∞ (H tak terhingga) dalam perancangan tracking servo untuk antena radar. Dengan mengasumsikan bahwa jalur lintasan target radar dapat diprediksi untuk beberapa waktu pencuplikan ke depan, maka informasi masa depan ini dapat digunakan untuk merancang pengendali traking yang memiliki sifat lebih kokoh terhadap gangguan dan memiliki kinerja tracking yang lebih baik. Pengendali preview-H_∞ pada makalah ini menggunakan umpan balik keadaan. Performansi tracking terhadap sinyal referensi telah dianalisa baik pada domain frekuensi maupun pada domain waktu. Responsi frekuensi telah dibandingkan dengan responsi frekuensi sistem kendali yang dirancang hanya berdasarkan pengendali H_∞, dan juga telah dibandingkan dengan sistem kendali yang dirancang hanya berdasarkan pengendali preview optimum konvensional. Untuk mengevaluasi responsi waktu, sinyal referensi berbentuk sinusoidal dan sinyal referensi berbentuk fungsi sigmoid telah digunakan. Dari hasil analisa numerik yang diperoleh, di makalah ini telah dibuktikan bahwa sistem kendali yang memakai pengendali preview-H_∞ memiliki kinerja tracking lebih baik daripada sistem yang dirancang hanya berdasarkan kendali H_∞, dan juga bahwa sistem tersebut lebih kokoh terhadap gangguan daripada sistem yang dirancang hanya menggunakan pengendali preview optimum.

Kata kunci: radar, antena, tracking, servo, umpan balik keadaan, pengendali, preview, H_∞.

1. Introduction

A tracking radar should follows the position of one or more objects in space, ignoring the content of the space not occupied by the target(s) being tracked. In Single Target Track (STT) the radar follows a single object and ignores all others. In Multi Target Track (MTT) the radar continuously monitors the position of several targets, with each target sampled many times per second. Effectively, one radar performs the function of as many tracking radars as there are targets being tracked. Multi-target

ISBN.9-793-68869-6

tracking requires that the antenna's beam position be changeable essentially instantaneously, and this is normally possible only with electronically scanned antennas. For true tracking to take place, the target(s) must be sampled at the Nyquist rate for the track servo bandwidth and target maneuvering bandwidths [1]. Figure 1 shows an illustration of travel path of radar target.

By the use of the so called $\alpha - \beta$ filter whose parameters are tuned using the Kalman filter algorithm the radar target travel path can be predicted [2]. When the radar target travel path undergoes totally unpredictable high-G maneuvers it is often desired to use multiple tracking gates [3].

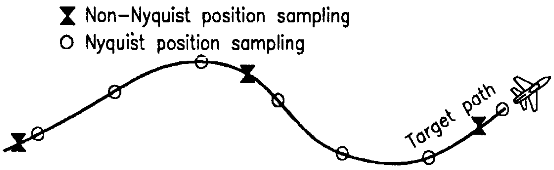

Figure 1. Travel Path of Radar Target [1].

By assuming that the radar target future travel path can be estimated for certain number of sampling period during operation, the future information of the reference signal can be utilized to construct a tracking control system with better tracking performance. The control method which incorporates the future information is well known as preview control. It has been known that, compared with feedback LQR/LQG, the preview control makes further reduction of the minimum value achievable by the cost function [4]. In spite of these advantages, the preview control has the lack in that it does not take into consideration the effects of un-certain disturbance.

The disturbances may be present in two types, i.e., internal disturbances due to un-modeled dynamics or parametric variation, and external disturbances such as friction forces, wind force and so on. Using H_∞ control theory, these disturbances can be taken into account in the design stage of the control system. However, the feedback H_∞ control uses only information corresponding to the present state value, and the limitations such phase-delay will probably be encountered.

This paper presents a design method of radar antenna tracking servo using state feedback robust tracking controller based on a novel control approach named preview- H_∞ control [5]. First, radar tracking system is briefly reviewed and a dynamical model of servo mechanism is formulated. Second, the procedure to design a preview-H_∞ controller is detailed. Third, formulae of the frequency response is derived, and the frequency responses are plotted and analyzed. Finally, the benefits and limitations of the tracking control using the preview-H_∞ controller are discussed.

2. Radar Tracking System Model

Figure 2 shows an illustration of a radar tracking system.

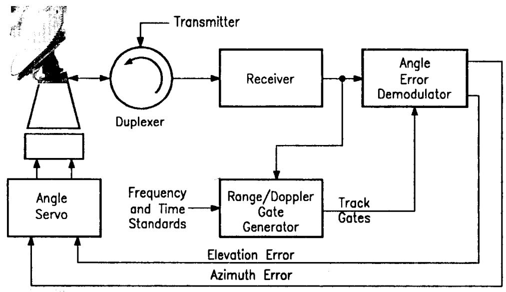

Figure 2. Radar Tracking System [1]

The system uses information derived in the antenna (receiver) and tracking circuits to move the antenna beam so that its axis position be in the direction of the target. The angle servo mechanism receives elevation error and azimuth error signals from the angle error demodulator and produces control input to the actuator.

Taking into account the stiffness of the mechanism and the dynamics of electrical motor, the dynamic equation of the radar antenna tracking servo can be formulated as follows,

$$J_o \frac{d^2\theta}{dt} + \left\{K_{cm} + \frac{K_t K_e}{R_a}\right\}\frac{d\theta}{dt} + K_{sp}(\theta - \theta_b) = \frac{K_t K_{pa}}{R_a}u \qquad (1)$$

$$J_p \frac{d^2\theta_b}{dt} + K_{cb}\frac{d\theta_b}{dt} + K_{sp}(\theta_b - \theta) = 0 \qquad (2)$$

J_o, J_p are the moment of inertia of the hub and the antenna payload, respectively. θ is the rotational angle of the hub, and θ_b is that of the antenna payload. K_{cm}, K_{cb} are the damping coefficients of the motor and of the antenna. K_t is the torque constant, K_e is the back electro motive force constant, and R_a is the internal resistance of the DC motor. K_{sp} is the flexural rigidity of the mechanism, K_{pa} is the gain of the power amplifier, and u is the control voltage.

Table 1 shows an example of parameters values set used in the numerical analysis.

Table 1. Servo Mechanism Parameter Values

Parameter	Value	Unit
Antenna-payload		
J_o	1.61 x 10^{-3}	$N\ ms^2/rad$
J_p	9.63 x 10^{-3}	$N\ ms^2/rad$
M	0.2	*Kg*
K_{cb}	9.32 x 10^{-3}	*N ms/rad*
K_{sp}	2.98 x 10^{-3}	*N ms/rad*
Motor-Hub		
R_a	3	Ω
K_t	0.0735	$N \cdot m / A$
K_e	0.0735	$V \cdot s / rad$
K_{pa}	10	-

3. Design of Preview-H_∞ Controller [5]

3.1. Controller Design

To design a preview-H_∞ controller, firstly a generalized plant including the nominal plant of the antenna servo mechanism and weighting coefficients has been constructed as shown figure 3.

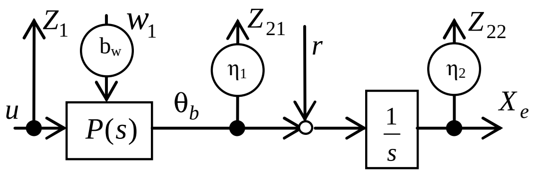

Figure 3. Generalized Plant

u is the control input, r is the reference signal, and w_1 is the torque disturbance. z_1, z_{21}, z_{22} denote the evaluation signals. z_1 is devoted to stability robustness while z_{21} and z_{22} are related to tracking performance specifications. Integral action has been incorporated after the error signal (r - θ_b) to ensure zero-steady-state error. b_w, η_1, η_2 are the weighting coefficients which should be selected properly to obtain a tracking control system with good tracking performance and robustness properties.

This generalized plant can be expressed in the following state-space equation,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} A & O \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_e \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} O \\ 1 \end{bmatrix} r + \begin{bmatrix} B_w \\ 0 \end{bmatrix} w_1 \quad (3)$$

$$\dot{x} = Ax(t) + B_1 u(t) + B_r r(t) + B_w w_1(t) \quad (4)$$

and the evaluation signals is given by,

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} O \\ C_2 \end{bmatrix} \begin{bmatrix} x \\ x_e \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad (5)$$

where $z_2 = [z_{21} \quad z_{22}]'$.

Here it is assumed that the reference signal $r(t)$ is known only during preview time, a certain interval of time τ, which is much lesser than the total working time T which is assumed to be infinity. The preview-H_∞ controller is designed to optimize the following cost function

$$J = \int_{t_0}^{t_0+\tau} [z(t)'z(t) - \gamma^2 w_1'(t) w_1 t] dt + \int_{t_0+\tau}^{\infty} [z(t)'z(t) - \gamma^2 w_1'(t) w_1 t] dt \quad (6)$$

where

$$w = \begin{bmatrix} r(t) \\ w_1(t) \end{bmatrix} \quad (7)$$

γ is a positive value which should be selected properly during the design process.

According to preview-H_∞ control theory, the control law which optimize the cost function is given by [5]

$$u(t) = -B_1' P x(t) - B_1' q(t) \quad (8)$$

where P>0 and q(t) are solutions of the following equations

$$PA + A'P + C_2'C_2 - P[B_1 B_1' - \gamma^{-2} B_w B_w'] P = 0 \quad (9)$$

$$\dot{q}(t) + [A - (B_1 B_1' - \gamma^{-2} B_w B_w')P]' q(t) + PB_r r(t) = 0 \quad (10)$$

where the terminal condition at the end of each preview time is

$$q(t + \tau) = 0. \quad (11)$$

It is important to note here, that if $\gamma = \infty$ then the preview control system becomes a conventional optimal preview control system.

3.2. Closed Loop System

Equation (9) has the form of Algebraic Riccati Equation (ARE) which is usually solved by means of a numerical method. While equation (10) should be solved backwards in time. From equation (10) the following equation is obtained

 ISBN.9-793-68869-6

$$q(t+\tau) = e^{-M\tau}q(t) + \int_t^{t+\tau} e^{-M(t+\tau-\delta)} PB_r r(\delta)\, d\delta, \quad (12)$$

where $M = \left[A - (B_1 B_1' - \gamma^{-2} B_w B_w')P\right]'$. Substituting the terminal condition of equation (11), the following equation is obtained

$$q(t) = e^{M\tau} \int_t^{t+\tau} e^{-M(t+\tau-\delta)} PB_r r(\delta) d\delta, \quad (13)$$

Transformation of variable $\varepsilon = \delta - t$ yields,

$$q(t) = e^{M\tau} \int_0^{t} e^{-M(t-\varepsilon)} PB_r r(\varepsilon + t) d\varepsilon. \quad (14)$$

The information about the reference signal is integrated along the preview time τ . By selecting proper sampling time $\Delta\varepsilon$, $q(t)$ in equation (14) can be approximated by the following expression,

$$q(t) = F_f \cdot r, \quad (15)$$

where,

$$F_f = \left[PB_r \quad e^{M\Delta\varepsilon} PB_r \quad \ldots \quad e^{(n-1)M\Delta\varepsilon} PB_r\right]$$

$$r = \left[r(t) \quad r(t+\Delta\varepsilon \quad \ldots \quad r(t+(n-1)\Delta\varepsilon\right]'$$

$$n = \frac{\tau}{\Delta\varepsilon}$$

Therefore, the control input is calculated as follows

$$u = -B_1' P x(t) - B_1' F_f r = u_b + u_f. \quad (16)$$

The closed loop system is obtained by substituting equation (16) into equation (4)

$$\dot{x} = (A - B_1 B_1' P)x - B_1 B_1' q + B_r r + B_w w_1 \quad (17)$$

Figure 4 shows the block diagram of the closed loop system.

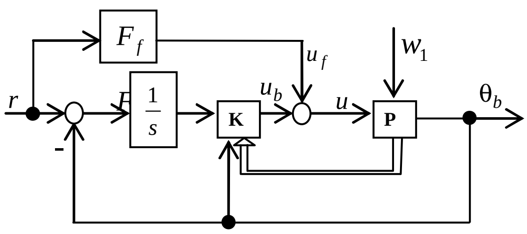

Figure 4. The Closed Loop System

4.Simulation Results

4.1.Frequency Response

The solution of equation (10) can also be represented by

$$q(t) = \int_t^{t+\tau} e^{(A_{cl}' + N')(\delta - t)} PB_r r(\delta) d\delta, \quad (18)$$

where $A_{cl} = A - B_1 B_1' P, \quad N' = \gamma^{-2} B_w B_w' P.$

Transformation of variables $\varepsilon = \delta - t$ yields

$$q(t) = \int_0^{\tau} e^{(A_{cl}' + N')(\varepsilon)} PB_r r(\varepsilon + t) d\varepsilon. \quad (19)$$

Therefore, the transfer function from r to q is

$$\begin{aligned} T_{qr}(s) &= \frac{Q(s)}{R(s)} = \int_0^{\tau} e^{(sI + A_{cl}' + N')\varepsilon} PB_r d\varepsilon \\ &= [sI + A_{cl}' + N']^{-1}\left[e^{(sI + A_{cl}' + N')\tau} - I\right]PB_r \\ &= e^{s\tau}[sI + A_{cl}' + N']^{-1}\left[e^{(A_{cl}' + N')\tau}\right]PB_r \\ &\quad - [sI + A_{cl}' + N']^{-1} PB_r \\ &= q_1(s) + q_2(s). \quad (20) \end{aligned}$$

On operating Laplace transform to the closed loop system of equation (17) the following equation is obtained

$$\begin{aligned} X(s) &= [sI - A_{cl}]^{-1} B_r R(s) \\ &\quad + [sI - A_{cl}]^{-1} B_1 \left(-B_1' T_{qr}(s)\right) R(s) \\ &\quad + [sI - A_{cl}]^{-1} B_1 w_1(s). \quad (21) \end{aligned}$$

The transfer function from the reference signal to the state vector is then given by

$$\frac{X(s)}{R(s)} = FB(s) + FB_0(s) \cdot [FF_1(s) + FF_2(s)], \quad (22)$$

where

$$FB(s) = [sI - A_{cl}]^{-1} B_r, \quad FF_1(s) = -B_1' q_1(s)$$

$$FB_0(s) = [sI - A_{cl}]^{-1} B_1, \; FF_2(s) = -B_1' q_2(s).$$

Or simply

$$\frac{X(s)}{R(s)} = FB(s) + FF(s). \quad (23)$$

$FB(s)$ and $FF(s)$ denote the transfer function of the feedback parts and the feed forward part.

To analyze tracking performance and robustness properties, certain frequency responses have been evaluated. The weighting coefficients have been selected as $b_w = 1, \quad \eta_1 = 10000,$ and $\eta_2 = 100000$.

Figure 5 shows the gain and phase of frequency respons $\Theta_b(s)/R(s)$, from reference signal $r(t)$ to the rotational angle of the antenna $\theta_b(t)$, of the control systems with feedback H_∞ controller and with preview- H_∞ controller (feedback + feed forward). The free parameter γ has been set as γ = 4.1 and the preview time is 2 second. It can be noted that in the low frequency domain where the gain of the both control system are the same, preview- H_∞ controller provides the advantage in that it keeps the phase of the control system be zero.

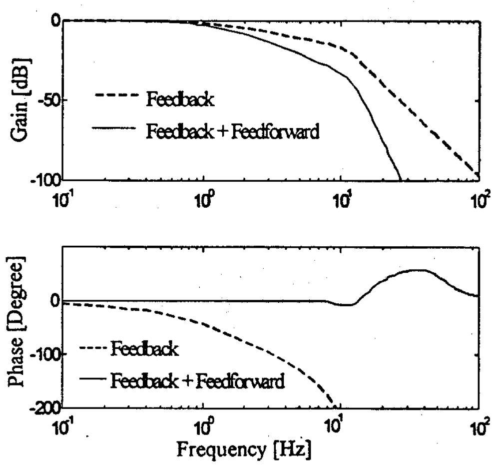

Figure 5. Frequency Responses of $\Theta_b(s)/R(s)$ of System with Feedback + Feed forward Controller.

Figure 6 shows the gain and phase of frequency response of $\Theta_b(s)/R(s)$ of the control systems with preview- H_∞ controller and with preview- H_2 controller. The preview- H_2 is designed by setting $\gamma = 10^{200}$ (nearly infinity). And figure 7 shows the gain of transfer function from disturbance $w_1(t)$ to $\theta_b(t)$ of those systems. From figure 6, it can be said that the preview- H_2 controller provides somewhat better tracking performance than the preview- H_∞ controller. However, from figure 7, it is clear that preview- H_∞ controller has better robustness properties.

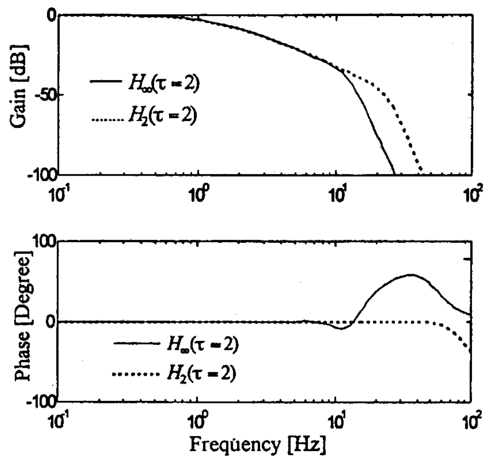

Figure 6. Frequency responses of $\Theta_b(s)/R(s)$ of control systems with preview- H_∞ controller and with preview- H_2 controller.

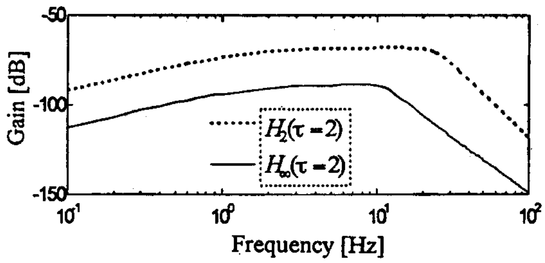

Figure 7. Gain of $\Theta_b(s)/w_1(s)$ of control systems with preview- H_∞ and preview- H_2 controllers.

4.2. Time Responses

To examine the tracking performance, two types of reference signal have been applied, i.e., sinusoidal signal and sigmoid-like signal. Here the weighting coefficients have been selected as the same as in the previous sub-subsection, and the sampling time is 1 millisecond. Figure 8 shows the rotational angle of the antenna $\theta_b(t)$ and total control input $u(t)$ when a sinusoidal signal $r(t)$ with the period of time of 2 second is applied to the control system with only a feedback H_∞ controller which is designed by setting $\gamma = 10$. It can be noted that the phase-delay exists between $r(t)$ and $\theta_b(t)$. During the steady state the control input is within the range of (-0.1 ~ 0.1) volt, while the maximum control input required at the transient stage is 0.56 Volt.

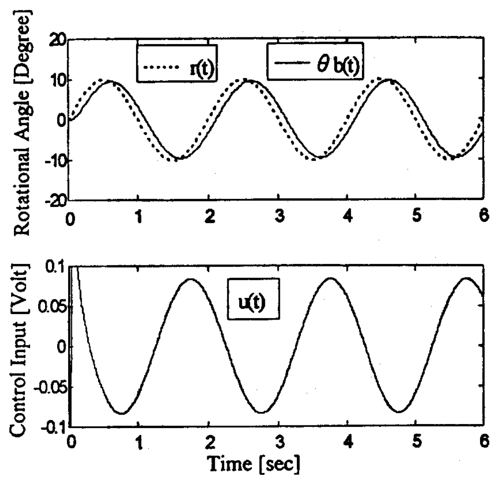

Figure 8. Sinusoidal Response of Control System with Feedback H_∞ controller.

Figure 9 shows the same response in which the preview- H_∞ controller with preview time of

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2 second is used. In the steady state, the phase-delay does not exist any longer and the control input is also within the range of $(-0.1 \sim 0.1)$ Volt. The maximum control input at the transient stage is 654 Volt. Figure 10 shows time responses when a sigmoid-like reference signal is applied. It is clear that the control system with preview-H_∞ controller response faster than that with only feedback H_∞ controller.

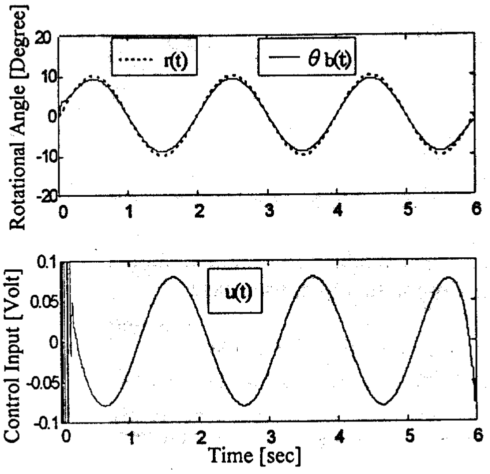

Figure 9. Sinusoidal Response of Control System with Preview-H_∞ controller.

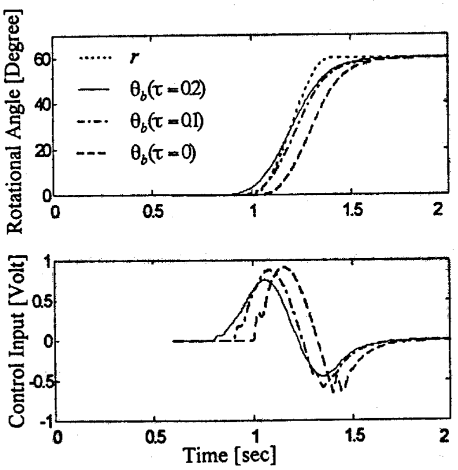

Figure 10. Response of Control System Towards Sigmoid-like Signal.

To observe the robustness properties, a step-like torque disturbance of 1 Nm is applied as $w_1(t)$. Figure 11 shows the response of $\theta_b(t)$ which demonstrates that the control system with preview-H_∞ controller has better disturbance attenuation than that with preview-H_2 controller.

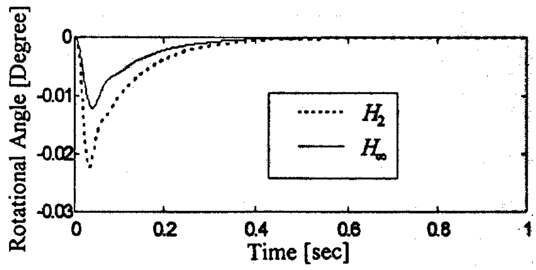

Figure11. Response Against a Step Disturbance.

5. Conclusion and Discussion

A radar antenna tracking servo has been designed using state feedback robust tracking controller based on preview-H_∞ control approach. The tracking performance and robustness properties have been compared with control system designed based on feed back H_∞ control as well as optimum preview control. Numerical analysis results have verified that preview-H_∞ controller is superior to feed back H_∞ controller in that it keeps the phase of the closed system be zero. Also it has been confirmed that preview-H_∞ controller has better disturbance attenuation capability over preview-H_2 controller (optimum preview controller).

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