Control of Substrate Concentration in a Fed-Batch Fermentor Using Robust $\mathcal{H}_\infty$ Controller

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Abstract

Fermentation is widely used in agri-biochemical processes. This paper presents a new method for designing a control system of growth limiting substrate concentration in a fed-batch fermentor. The control system design method utilizes feedback linearization and robust $\mathcal{H}_\infty$ controller. Assuming that growth can be modeled in a proper formulae, a feeding policy which ensures optimum grow rate in a fed-batch fermentor can be formulated. However, when disturbances affect the fermentation process the feeding policy can lead to considerable error. To cope with this problem in this paper a feedback controller is developed. The feedback controller and the feeding policy are used together to construct a control system which can make the concentration of the growth limiting substrate follow an *optimum concentration trajectory* even under disturbances. The feedback controller is developed based on feedback linearization coupled with robust $\mathcal{H}_\infty$ controller. Simulation results show that the controller proposed in this study enhances the performance considerably.

Keywords—Fed-Batch, Fermentor, Growth Limiting Substrate Concentration, Feeding Policy, Feedback Linearization, Robust, $\mathcal{H}_\infty$ Controller

1 Introduction

Production of Single Cell Protein (SCP) is the highest method to provide the need of protein in *feed and food*. Raw material containing rich carbon sources (glucose etc.) and other nutrient is converted into cells of micro-organism in a bioreactor. Generally, cells are cultivated in batch-culture to provide an appropriate cell number followed by fed-batch culture. To optimize productivity, nutrient is fed into fermentor intermittenly preventing inhibition by substrate formation of side-product. Therefore, concentration control of the substrate is necessary to maintain cell grow rate optimum throughout cultivation.

The objective of this study is to design a control system used in a fed-batch fermentor which has the following properties:
(1) Attenuates effect of disturbances, including parameters variation etc., on the quality of the product,
(2) Optimizes the productivity by minimizing time required in the fermentation process.

2 Dynamical Model of Fermentation Process

A fed-batch fermentation process shown in Figure 1 is considered. This fermentation process produces cell mass of concentration $X[g/L]$. As the simplest case, suppose that the growth of micro-organism in the fed-batch fermentor vessel is limited by a single nutrient, the growth limiting substrate, which in this case is glucose. The reaction in the fermentor vessel is

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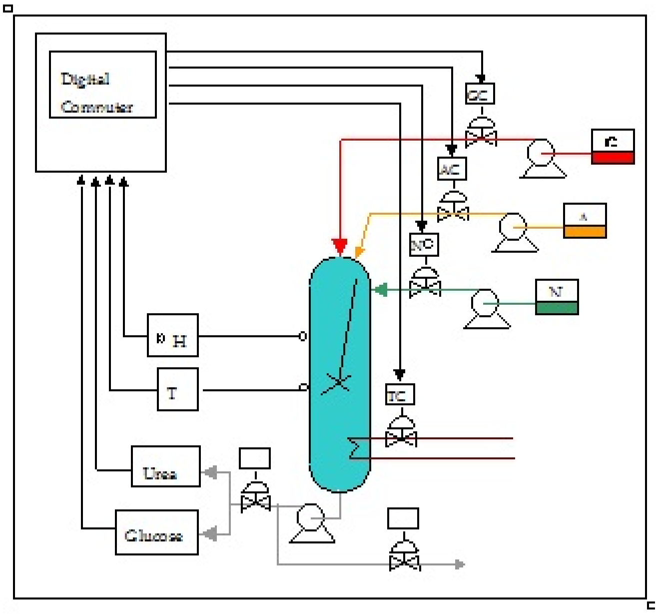

Figure 1: Structure of a Fed-Batch Fermentation Process

governed by the following relation:

$$Glucose(C,H,O) + Amonia(N,H) + O_2 \longrightarrow Yeast(C_mH_nO_oN_p) + CO_2, \tag{1}$$

where the concentration of Glucose and Yeast are denoted by $S[g/L]$ and $X[g/L]$. Here, Amonia and Oxigen contribute at small amount which can be neglected. The substrate is fed into the fermentor vessel with inlet flow rate $F[L/h]$ in which the constant concentration $S_f[g/L]$ of the growth limiting substrate is included. Based on balance of cell mass, substrate, and culture broth volume, we obtain the following differential equations [1]:

$$\frac{dX}{dt} = -\frac{F}{V}X + \mu(s)X, \quad X(0) = X_o \tag{2}$$

$$\frac{dS}{dt} = -\frac{\mu(s)}{Y}X + \frac{S_f - S}{V}F, \quad S(0) = S_o, S_f > 0 \tag{3}$$

$$\frac{dV}{dt} = F, \quad V(0) = V_o \tag{4}$$

where Y is the cell yield (dimensionless). Since we have the following stoichiometri relation

$$X = Y(S_f - S) \tag{5}$$

the state equations become

$$\frac{dS}{dt} = -\mu(s)S_f + \mu(s)S + \frac{S_f - S}{V}F, \ (S_o = S_e) \tag{6}$$

$$\frac{dV}{dt} = F \tag{7}$$

where S_e denotes the growth limiting substrate concentration at the end of the previous cycle, and S_o denotes that at the beginning of the next cycle.

In this study the micro-organism are described by substrate inhibition kinematics as follows:

$$\mu(s) = \frac{\mu_m S}{K_m + S + S^2/K_i} \tag{8}$$

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where μ_m, K_m, K_i are parameters obtained experimentally [1]. Thus the dynamical model of the fermentation process can be expressed by the following *non-linear differential equation*:

$$\frac{dS}{dt} = \frac{-S_f \mu_m S + \mu_m S^2}{K_m + S + S^2/K_i} + \frac{S_f - S}{V} F \tag{9}$$

$$\frac{dV}{dt} = F \tag{10}$$

3 Controller Design

Figure 2 shows the control structure proposed in this study.

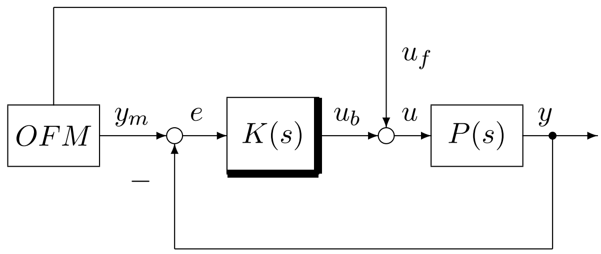

Figure 2: Complete Control System Structure

P is the fermention process to be controlled, K is a feedback controller, and OFM stands for Optimum Feeding Mechanism. The OFM is intended to provide the necessary feed for following the specified substrate concentration trajectory S_n. The feedback controller then stabilizes the tracking error dynamics by attenuating the effect of unknown disturbances.

From the result of the optimum feeding policy [2][3], it is assured that the following assumptions hold

$$\left.\begin{array}{l} S_f >> S \\ V \neq 0 \end{array}\right\} \tag{11}$$

Therefore from equation (9), we can conduct the following feedback linearization [5][6]:

$$\dot{S} = \psi \tag{12}$$

$$u_b = g(S,V)^{-1}\{\psi - f(S)\} \tag{13}$$

where $\dot{S} = \frac{dS}{dt}, u_b = F, g(S,V) = \frac{S_f - S}{V}, f(S) = \frac{-S_f \mu_m S + \mu_m S^2}{K_m + S + S^2/K_i}$.

Now, we want to design a robust feedback controller based on the linearized plant in (12). Here a robust $\mathcal{H}_\infty$ controller is designed based on Linear Matrix Inequality approach [4]. Recalling the objective of this work and from the contol system structure shown in figure 2, a generalized plant shown in figure 3 is considered.

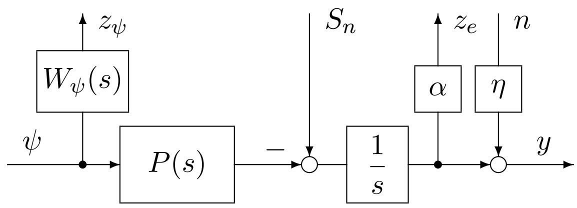

Figure 3: Generalized Plant $G(s)$

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$P(s)$ denotes the nominal plant given in (12), and $W_\psi(s)$ is a weighting function to limit the control action ψ as well as to provide robust stability against high frequency disturbances. The integral action is introduced to ensure that the steady state error between the optimal concentration trajectory S_n and the real concentration trajectory S_r is zero; $e = S_n - S_r$. α is a weighting coefficient to adjust optimal achievable tracking performance, while η is a substantially small scalar which is incorporated in order to satisfy the assumption required in designing $\mathcal{H}_\infty$ controllers based on the LMI approach.

The weighting function $W_\psi(s)$ is given by

$$W_\psi(s) = K_\psi \frac{s+\omega_1}{s+\omega_2} \Longrightarrow \left\{ \begin{array}{lcl} \dot{x}_\psi & = & A_\psi x_\psi + B_\psi \psi \\ z_\psi & = & C_\psi x_\psi + D_\psi \psi \end{array} \right. \tag{14}$$

then we have the following state-space equation and output equation of the generalized plant

$$\left[\begin{array}{c} \dot{S} \\ \dot{e} \\ \dot{x}_\psi \\ \hline z_\psi \\ z_e \\ \hline y \end{array} \right] = \left[\begin{array}{ccc|cc|c} 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_\psi & 0 & 0 & B_\psi \\ \hline 0 & 0 & C_\psi & 0 & 0 & D_\psi \\ 0 & \alpha & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & \eta & 0 \end{array} \right] \left[\begin{array}{c} S \\ \int edt \\ x_\psi \\ \hline S_n \\ n \\ \hline \psi \end{array} \right] \Longrightarrow \left\{ \begin{array}{lcl} \dot{x} & = & Ax + B_1 w + B_2 \psi \\ z & = & C_1 x + D_{11} w + D_{12} \psi \\ y & = & C_2 x + D_{21} w + D_{22} \psi \end{array} \right. \tag{15}$$

where $x = [S \quad \int edt \quad x_\psi]^T$, $w = [S_n \quad n]^T$, and $z = [z_\psi \quad z_e]^T$.

To achieve the objective, a robust $\mathcal{H}_\infty$ controller K_b is designed in order that the transfer function matrix from the exogenous input signal w to the evaluation signal z satisfies the following $\mathcal{H}_\infty$ norm criterion:

$$||T_{zw}||_\infty < 1 \tag{16}$$

Because of the integral action inside the generalized plant, the feedback controller K in figure 2 is given as follows:

$$K(s) = \frac{K_b(s)}{s}. \tag{17}$$

4 Results

Figure 4 shows the flow chart of OFM [2]. The feedback controller has been designed by setting the parameters values as $K_\psi = 31.62, w_1 = 2.51, w_2 = 79477, \alpha = 10$, and $\eta = 0.000001$. After combining the OFM and the feedback controller to construct a complete controller, the controller has been applied to a fermentor where there exists a parameter variation $\bar{\mu}_m = \mu_m(1+75\%)$. Parameters values used in computer simulation are $\mu_m = 0.53[1/h], K_m = 1.2[g/L], K_i = 22[g/L], Y = 0.4$, and $S_f = 20[g/L]$. The initial volume, V_0, and the final volume, V_e are assumed to be 2 and 10 L, respectively. The initial cell-mass concentration, X_0 is assumed to be 8 (g/L) and the initial substrate concentration, S_0, is assumed to be 0 (g/L).

The simulation has been carried out at the sampling time of 2 second. Figure 5(a)shows substrate concentration. The dotted line is the reference trajectory S_n, the broken line is the substrate concentration trajectory without feedback controller, and the solid line is the substrate concentration trajectory with the designed $\mathcal{H}_\infty$ feedback controller. It can be noted that even though μ_m varies during the time 90 minute to 100 minute, the substrate concentration has been successfully maintained optimum by the use of the proposed controller. Figure 5(b) shows the corresponding cell concentration. Again the proposed controller gives good performance.

Figure 6(a) shows the inlet flow to the fermentor vessel. The broken line is the reference inlet flow calculated by the OFM. The inlet flow without the feedback controller is just the same as this reference inlet flow. The solid line denotes the inlet flow when the feedback controller

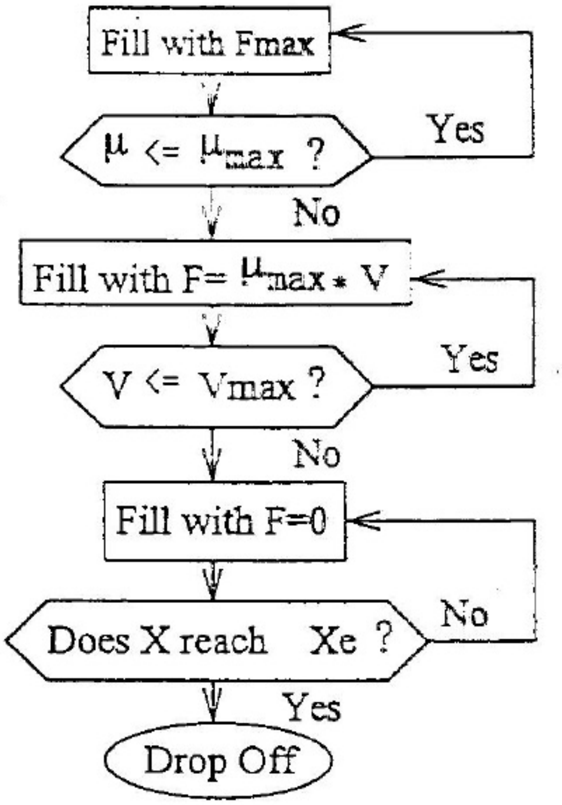

Figure 4: OFM Flow Chart

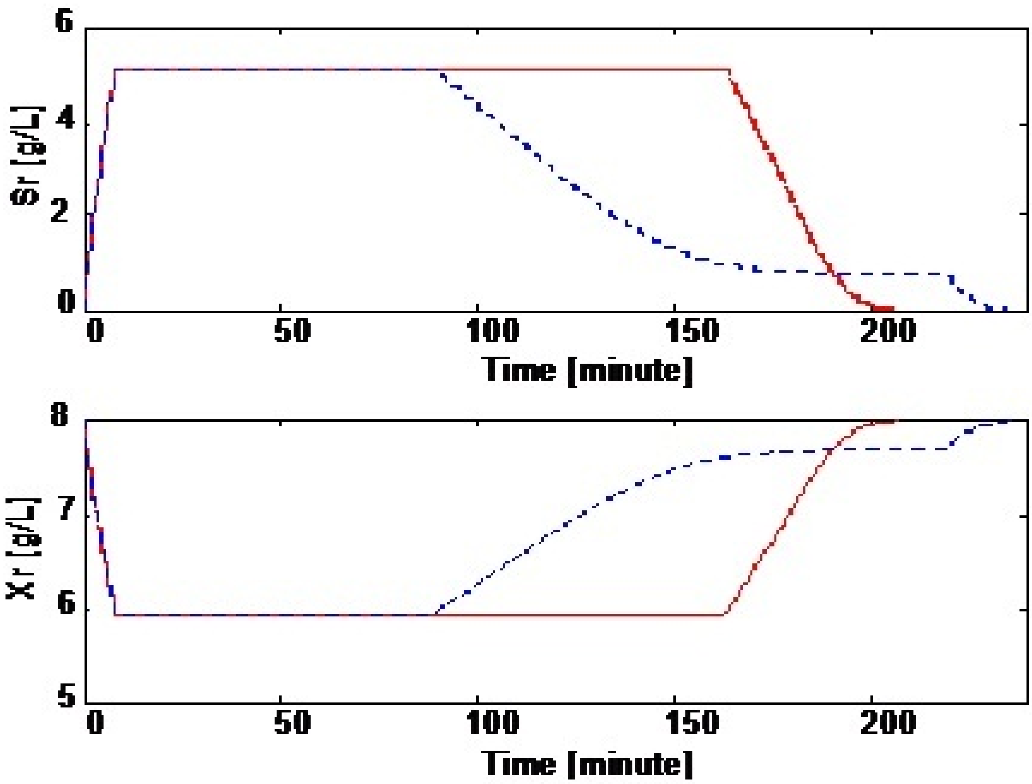

Figure 5: Substrate Concentration S_r and Cell Concentration X_r. (—: with the feedback controller; – – – : without any feedback controller; - - - : optimum trajectory (reference))

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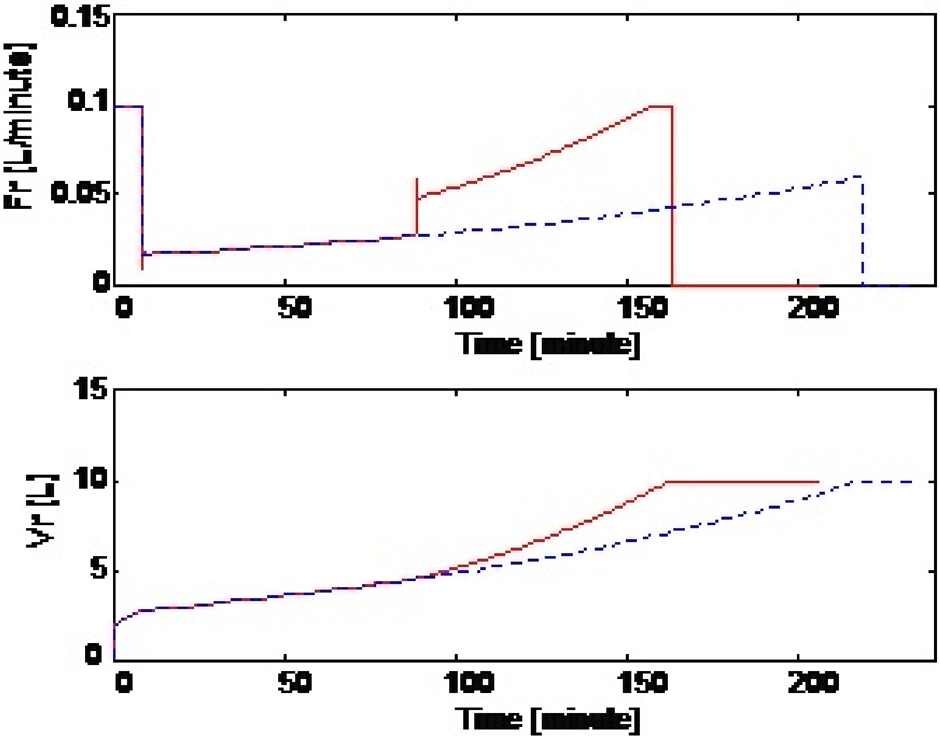

Figure 6: Inlet Flow F_r and Volume of Broth in the Fermentor Vessel V_r. (——: with the feedback controller; – – – : without any feedback controller)

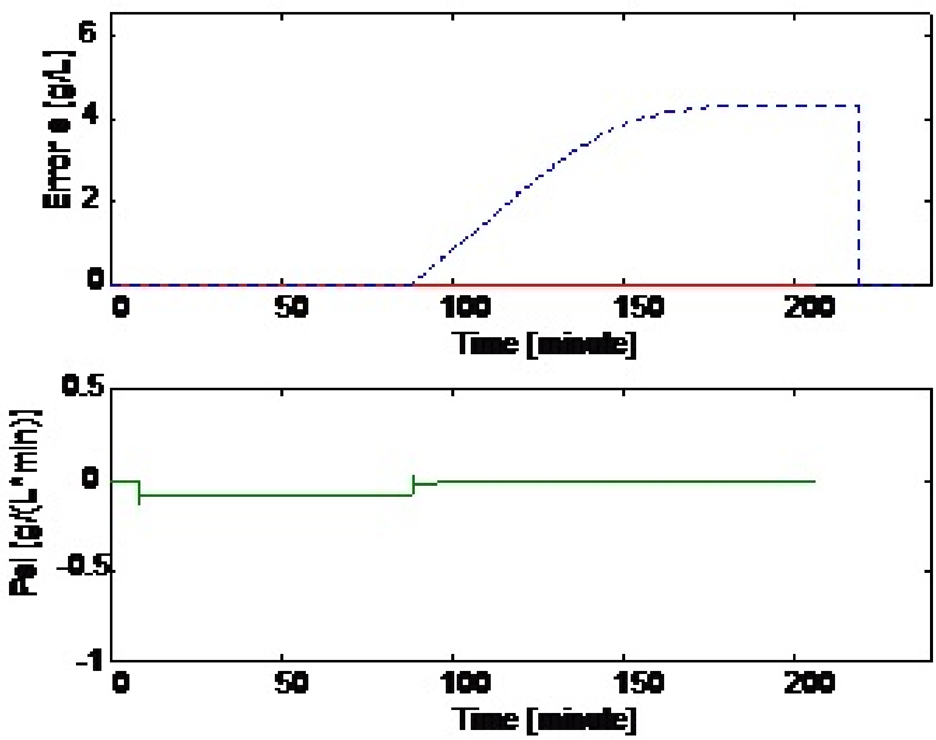

Figure 7: Error $e(t) = S_n(t) - S_r(t)$ and ψ

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is used. When the parameter variation occurs the feedback controller changes inlet flow to compensate it. Figure 6(b) shows the corresponding volume of the vessel.

Figure 7(a) shows the error between reference substrate concentration S_n and substrate concentration in the reactor, S_r. The broken line is the result of the control system without the feedback controller. The solid line is the result of the control system equipped with the feedback controller. It is obvious that the feedback controller functions in maintaining the error be zero. Figure 7(b) shows the inlet flows contributed by the feedback controller. The solid line denotes u_b given in equation (13), while the broken line denotes the feedback control input of the linearized plant ψ in equation (12).

5 Conclusion and Discussion

The feedback controller effectively attenuates the effect of the parameter variation. This results in a satisfactory tracking performance; the substrate concentration S_r follows accurately the *optimum substrate concentration* S_n. The substrate concentration S_r is directly measured using a glucose bio-sensor so that this control scheme may be applied to other fermentation processes having different model of substrate inhibition kinematics. In the proposed control method, only the substrate concentration S_r is controlled while the vessel volume V_r is left free. The maximum value V_e has been set as the boundary condition, so that when V_r reaches V_e, both feedforward feeding u_f and the feedback control input u_b are stopped according to the OFM procedure.

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