Robust Control of Composition in a Binary Distillation Column:
Robustness Against Actuator Gain Variation and Time Delay

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Abstract
Distillation plays significant role for separation and purification in bio-chemical processes. This paper addresses a novel design method of robust composition control system of a high purity binary distillation column. An LV-configuration which uses reflux (L) and boilup (V) as manipulated inputs is considered. A distillation column, which is a Multi-Input-Multi-Output (MIMO) system and has complexed dynamics, is difficult to be controlled. It is also subject to disturbances due to unmodeled dynamics, parameters variations, and actuators characteristics uncertainties. To ensure that the control system is stable even under such uncertainties and to maintain the quality of distillate and bottom products, a robust control system is necessary. In this paper, first, a linearized dynamic model of a binary distillation column and control problem formulation are described. Next, a robust controller is designed based on a nominal plant using $\mu$ synthesis. Performance of the controller is evaluated through numerical experiments. The design controller is used to control composition in a pseudo-real plant where control valves have gain uncertainties and time delay. Simulation results show that the designed controller provides robustness properties and satisfies well a given performance specification.

Keywords—Robust control, $\mu$ synthesis, High purity distillation column, LV-configuration, Nominal plant, Psudeo-real plant.

1 Introduction

In bio-chemical processes, distillation which exploits the difference in the boiling points of multicomponent liquids, plays important roles for separation and purification. For example, a certain number of distillation columns are used in purification stage after fermentation stage in an ethanol plant [1]. To keep the quality and to produce the product efficiently with low cost, it requires to be controlled.

Consider a binary-distillation column shown in figure 1 [3][4][7]. From a control point it may be viewed as a Multi-Input-Multi-Output (MIMO) system or in this case $5 \times 5$ system. Based on all available information such as: measurements, process model, and expected disturbance, a controller should manipulate all 5 inputs $(L, V, V_T, D, B)$ in order to maintain the 5 outputs (levels in top and bottom, pressure, top and bottom composition) as close as possible to their desired values. Yet, to simplify the controller design, to make failure tolerant, and to get a controller which is insensitive to plant operation, a Decentralized Control System (DCS) based on single loops is used in practice. Since the levels and pressure have to be controlled at all times to ensure stable operation, and because these control loops are essentially independent of the composition control, the level and pressure control system is designed first.

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2 Modeling and Problem Formulation

In this section a dynamic model of high purity binary distillation columns is described \cite{3}\cite{4}\cite{7}. In order to simplify the problem, the following assumptions are made: (1) binary separation, (2) constant relative volatility, (3) constant molar flows, (4) constant liquid holdup on all trays, (5) no vapor holdup, (6) Vapor-Liquid-Equivalent (VLE) and perfect mixing on all trays, (7) perfect pressure and level control. The last assumption results in immediate flow response, that is, the flow dynamics is neglected. This neglected flow dynamics is incorporated in the controller design procedure as uncertainty.

Based on material balances for change in holdup of light component, we get a nonlinear dynamic model. By linearizing the material balance on each tray, we obtain a linearized dynamic model written in the state-space equation in terms of deviation variables,

\[ \begin{align*}
\dot{x}_l &= A_l x_l + B_l u_l \\
y_l &= C_l x_l
\end{align*} \tag{1} \]

where \( x_l = [dx_1, \ldots, dx_{N+1}]^T \) are the tray compositions, \( u_l = [dL, dV]^T \) are the manipulated inputs and \( y_l = [dy_D, dx_B]^T \) are the controlled outputs.

By Laplace transform, from the state-space equation in (1) we have

\[ \begin{bmatrix} dy_D(s) \\ dx_B(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} dL(s) \\ dV(s) \end{bmatrix} \tag{2} \]

Each element of this transfer function matrix has the order of 41. In designing a controller, low order controller is preferable. Taking into consideration only the first 6 dominant time constants, the plant order is then reduced from 41st order to 6th order. This reduced order plant is referred to as nominal plant \( P_N(s) \). It has already checked through computer simulation that up to frequency of 0.2 [1/min], the reduced order model represents well the full order plant dynamics.
However, to be applicable in practice, the following matters are of high importance and have to be taken into account:

1. The fluid dynamics should be taken into account
2. Actuators gain variations may exist and actuators always have time delay.

Figure 2 shows the block diagram of the nominal plant with such uncertainties. The distillate composition $y_1$ and the bottom product composition $y_2$ are the controlled outputs. The reflux $u_1$ and the boil-up $u_2$ are manipulated inputs whose flow rates are manipulated by actuators to control $y_1$ and $y_2$. $d$ represents disturbances.

Figure 3 shows closed loop control system constructed by the pseudo-real plant $P$ (distillation column with full order dynamics, having actuators gain variations, time delay, and other unknown disturbances) and a feedback controller $K(s)$.

The most essential objective to be achieved is that the closed loop system has to be stable even under presence of external disturbances and internal disturbances including model-plant mismatch (uncertainty). This specification is called "Robust Stability (RS)" specification. For example, assume there is uncertainty with respect to the actual magnitude of the manipulated inputs (see figure 2). The perturbed plant, $P_p$, is then $P_p = PN(I + \Delta_d)$ where $\Delta_d(s)$ denotes the relative uncertainty. In this paper, we will consider the case when the magnitude of the uncertainty is equal for both inputs: $|\Delta_{di}| \leq |W_d(j\omega)|$, $i = 1, 2$. The robust stability is measured using complementary sensitivity function $T(s)$. In terms of $H_\infty$-norm the robust stability criterion is expressed as $\sup \sigma(W_d)T \leq 1$; $T = PNK(I + PNK)^{-1}$ where $K(s)$ and $W_d(s)$ denote controller and robust stability weighting function, respectively.

The next objective is to keep the controlled outputs be close to the desired setpoints. This is called "Regulating Performance " for constant setpoints, or "Tracking Performance" for the case when setpoints are changed with time. When we want to refer to both of these performances, it may be called just "Performance". This performance is measured using sensitivity function $S(s)$ of the closed loop system, so that the magnitude of the sensitivity function is less than an upper bound. The upper bound is the magnitude of the inverse of performance weighting function $W_p(s)$: $\sigma(S(j\omega)) < \frac{1}{|W_p(j\omega)|}, \ \forall \omega$; where $S = (I + PNK)^{-1}$. In terms of $H_\infty$-norm, this performance specification may equivalently be formulated as: $\sup \sigma(W_pS) \leq 1$. The weighting function $W_p(s)$ is then used to specify the frequency range over which the output errors are to be
small. The Robust Stability and Performance specifications can be coupled into one $H_\infty$-norm criterion as
\[
\sup_\omega \sigma(\Psi(s)) \leq 1 \iff ||\Psi(s)||_\infty \leq 1; \text{ where } \Psi(s) = \begin{bmatrix} W_d T & 0 \\ 0 & W_p S \end{bmatrix}.
\]
(3)

3. Robust Feedback Controller Based on $\mu$ Synthesis

Following the problem formulation in section 2, the specifications for a controller design are detailed as follows:
1. The design should allow for a time delay of one minute on the control input (control action) and for $\pm 5\%$ uncertainty in the actuator gains.
2. The final steady-state values of all variables ($y_D, x_B$) should be within $1\%$ of their desired values.
3. For distillate product composition $y_D$, where a change is demanded, the composition should be within $\pm 10\%$ of the desired final value within 40 minutes.
4. The above specifications should be satisfied for both the reduced order linear model and the full order linear model.

To realize the above specifications, a generalized plant shown in figure 4 is considered.

Figure 4: Generalized Plant for $\mu$ Synthesis

Here, $w = (w_1, w_2)^T$ are exogenous inputs, $z = (z_1, z_2)^T$ are evaluation signals, $u$ is control input, and $y$ is measured signal. The transfer function which maps $w$ to $z$ is
\[
\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} W_d T & W_d K S \\ W_p P_N S & W_p S \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}
\]
(4)
\[
z = T_{zw} w
\]
(5)

where $S$ and $T$ denote sensitivity function and complementary sensitivity function.

From figure 4 and equation (5) the magnitude bound of the uncertainty is formulated as
\[
W_d(s) = \text{diag}[W_{d1}(s), W_{d2}(s)], \quad W_{d1}(s) = W_{d2}(s) = 2\frac{s + 0.05}{s + 2}.
\]
(6)

Robust performance weighting function is selected as
\[
W_p(s) = \text{diag}[W_{p1}(s), W_{p2}(s)], \quad \text{where } \begin{cases} W_{p1}(s) = 0.5 \frac{5s+1}{5s+\xi} \\ W_{p2}(s) = 0.25 \frac{10s+1}{10s+\xi} \end{cases}
\]
(7)

$\xi$ is a substantially small value ($\xi = 10^{-6}$). This robust performance weighting function has been determined to impose strict requirement on the performance of the distillate product composition
$y_D$ in order to fulfill the 3rd specification.

A robust performance $H_\infty$ controller $K_\mu$ has been designed based on $\mu$ synthesis method [2][5][6]. The structured uncertainty is $\Delta_S = \text{diag}[\Delta_1, \Delta_2]$ where $\Delta_1$ and $\Delta_2$ denote a scalar block and a full block, respectively.

The robust performance controller $K_\mu(s)$ has been applied to the pseudo-real plant. Figure 5 shows time responses towards a series of different setting points around a certain operating point. The upper figure shows the responses of distillate composition $y_D$ and the bottom product composition $x_B$ ($y_D$: solid line, $x_B$: broken line), while the lower figure shows the manipulated inputs (control inputs) i.e., reflux $d_L$ and boilup $d_V$ ($d_L$: solid line, $d_L$: broken line). Initially, the distillation process is operated in a certain operating point. At time 10 minute, both distillate product $y_D$ and bottom product $x_B$ are increased at amount 0.01 mol/minute. The actuator gain of each manipulated input suddenly varies +20% around time 90 minute. At time 130 minute the setting point is changed to -0.01 mol/minute. The distillate product and bottom product are measured using appropriate sensors, and these values are used as feedback signals. During the operation, each sensor is affected by random noise whose magnitude is 10% of the setting point. It can be noted that the closed loop of the perturbed pseudo-real plant is kept stable, and it responds well with zero steady state error for both bottom product and distillate product, and with short settling time for distillate product $y_D$. These results exhibit that the given specifications have been achieved successfully.

4. Conclusion and Discussion

Based on $\mu$ synthesis, a robust performance $H_\infty$ controller has been designed using the nominal plant of order $6^{th}$. The controller has been implemented to the pseudo-real plant of distillation column in computer simulation, and it has been verified that the controller well satisfies the given design specifications.

Only perturbation due to the variation in input action and its time delay is considered in this study. Other perturbations may exist in a distillation process. Also, only a certain operating point has been considered in this study. For the case when operating point is often changed from one to another, the controller designed in this paper may be coupled with gain scheduling approach to solve such a problem.
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References


