μ SYNTHESIS BASED ROBUST CONTROLLER FOR COMPOSITION CONTROL OF BINARY DISTILLATION COLUMNS

Estiko Rijanto *

Venture Business Laboratory,
Tokyo University of Agriculture and Technology

International House Room 320, 2-41 Saiwaicho, Fuchushi, Tokyo 183-0054, Japan

Tel/Fax: +81-42-336-4955; e-mail: estiko@cc.tuat.ac.jp

Abstract

Distillation plays significant role for separation and purification in process industry. This paper is concerned with a design method of robust control system of high purity binary distillation columns using μ synthesis. The μ synthesis involves an iteration of determining optimal scaling matrix D and optimal $\mathcal{H}_\infty$ controller. The LV-configuration which uses reflux(L) and boilup(V) as manipulated inputs is considered. The distillation column has plant uncertainty such as: uncertainty due to parameters of the linearized model which are only known approximately, actuators characteristics uncertainty, and parameters variation due to nonlinearity and changing in the operating conditions. To ensure that the control system is stable under such uncertainties and to maintain the quality of the product, a robust control system is required. In this paper, first, a linearized dynamic model of a binary distillation column and control problem formulation are described. Next, a robust controller is designed based on μ synthesis. Performance of the controller is compared with that of optimal $\mathcal{H}_\infty$ controller through numerical experiments. It has been found that μ synthesis controller enhances the performance considerably, compared with optimal $\mathcal{H}_\infty$ controller.

Keywords— μ synthesis,robust controller,distillation column, LV-configuration,$\mathcal{H}_\infty$ controller.

INTRODUCTION

In process industry, distillation which exploits the difference in the boiling points of multicomponent liquids, plays important roles for separation and purification. To keep the quality and to produce the product efficiently with low cost, it requires to be controlled.

Consider a binary-distillation column shown in figure 1. From a control point it may be viewed as a Multi-Input-Multi-Output(MIMO) system or in this case 5×5 system. Based on all available information such as: measurements, process model, and expected disturbance, a controller should manipulate all 5 inputs (L, V, V_T, D, B) in order to maintain the 5 outputs (levels in top and bottom, pressure, top and bottom composition) as close as possible to their desired values. Yet, to simplify the controller design, to make failure tolerant, and to get a controller which is insensitive to plant operation, a Decentralized Control System(DCS) based on single loops is used in practice. Since the levels and pressure have to be controlled at all times to ensure stable operation, and because these control loops are essentially independent of the composition control, the level and

*On leave from LIPI, Indonesian Institute of Science

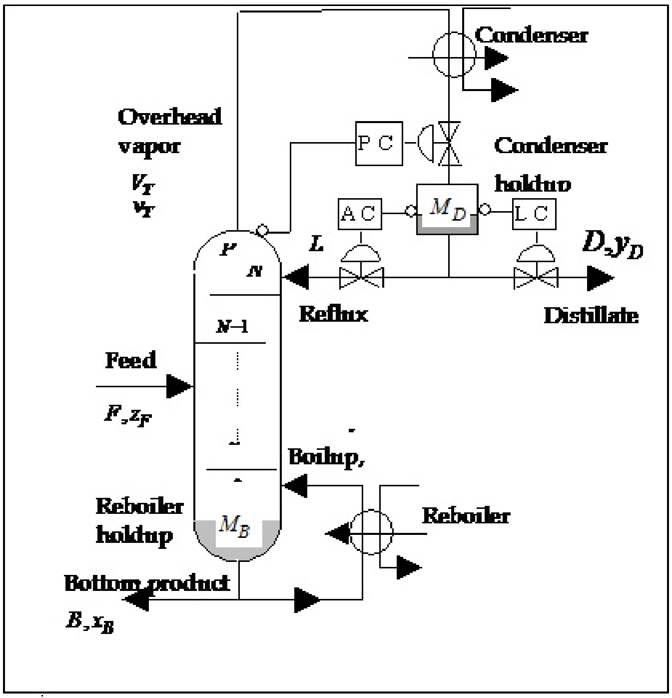

Figure 1: Binary-Distillation Column With Single Feed and Total Condenser

pressure control system is designed first.

DYNAMIC MODEL & CONTROL PROBLEM FORMULATION

In this section a dynamic model of high purity binary distillation columns is described [2][3]. In order to simplify the problem, the following assumptions are made:(1) binary separation, (2) constant relative volatility, (3) constant molar flows, (4) constant liquid holdup on all trays, (5) no vapor holdup, (6) Vapor-Liquid-Equivalent (VLE) and perfect mixing on all trays, (7) perfect pressure and level control. The last assumption results in immediate flow response, that is, the flow dynamics is neglected. This neglected flow dynamics is incorporated in the controller design procedure as uncertainty.

Based on material balances for change in holdup of light component, we get a non-linear dynamic model. By linearizing the material balance on each tray, we obtain a linearized dynamic model written in the state-space equation in terms of deviation variables,

$$\left\{ \begin{array}{lcl} \dot{x}_l & = & A_l x_l + B_l u_l \\ y_l & = & C_l x_l \end{array} \right. \tag{1}$$

where $x_l = [dx_i, \ldots, dx_{N+1}]^T$ are the tray compositions, $u_l = [dL, dV]^T$ are the manipulated inputs and $y_l = [dy_D, dx_B]^T$ are the controlled outputs.

By Laplace transform, from the state-space equation in (1) we have

$$\begin{bmatrix} dy_D(s) \\ dx_B(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} dL(s) \\ dV(s) \end{bmatrix} \tag{2}$$

Each element of this transfer function matrix has the order of 41. In designing a controller, low order controller is preferable. Taking into consideration only the first 6 dominant time constants, the plant order is then reduced from 41^{st} oder to 6^{th} order. This reduced order plant is referred to as nominal plant $P_N(s)$. It has already checked throught computer simulation that up to frequency of 0.2 [1/min], the recuded order model represents well the full order plant dynamics.

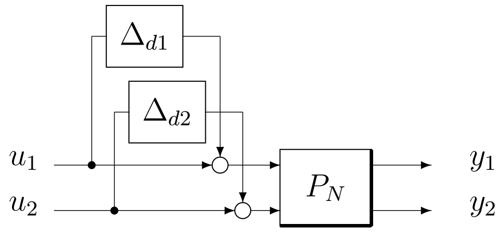

Figure 2: Nominal Plant P_N with Uncertainties

However, to be applicable to the real world, the following matters are of high importance and have to be taken into account:

1. The fluid dynamics should be taken into account
2. Actuators gain variations may exist and actuators always have time delay.

Figure 2 shows the block diagram of the nominal plant with such uncertainties.

The most essential objective to be achieved is that the slosed loop system has to be stable even under presence of external disturbances and internal disturbances including model-plant mismatch(uncertainty). This specification is called "Robust Stability (RS)" specification. For example, assume there is uncertainty with respect to the actual magnitude of the manipulated inputs (see figure 2). The perturbed plant, P_p, is then $P_p = P_N(I + \Delta_d)$ where $\Delta_d(s)$ denotes the **relative uncertainty**. In this paper, we will consider the case when the magnitude of the uncertainty is equal for both inputs; $\Delta_{di}| \leq |W_d(j\omega)|, \;\; i = 1, 2$. The robust stability is measured using complementary sensitivity function $T(s)$. In terms of $\mathcal{H}_\infty$-norm the robust stability criterion is expressed as $\sup_\omega \bar{\sigma}(W_d T) \leq 1; \quad T = P_N K(I + P_N K)^{-1}$ where $K(s)$ and $W_d(s)$ denote controller and robust stability weighting function, respectively.

The next objective is to keep the controlled outputs be close to the desired setpoints. This is called "Regulating Performance " for constant setpoints, or "Tracking Performance" for the case when setpoints are changed with time. When we want to refer to both of these performances, it may be called just "Performance". This performance is measured using sensitivity function $S(s)$ of the closed loop system, so that the magnitude of the sensitivity function is less than an upper bound. The upper bound is the magnitude of the inverse of performance weighting function $W_p(s)$: $\bar{\sigma}(S(j\omega)) < |\frac{1}{W_p(j\omega)}|, \;\; \forall\omega; \quad$ where $S = (I + P_N K)^{-1}$. In terms of $\mathcal{H}_\infty$-norm, this performance specification may equivalently be formulated as: $\sup_\omega \bar{\sigma}(W_p S) \leq 1$. The weighting function $W_p(s)$ is then used to specify the frequency range over which the output errors are to be small.

Most classical frequency domain specifications may be captured by this approach. For example, assume that the following specifications are given in the frequency domain: (1) Steady-state error less than ψ, (2) closed loop bandwidth higher than ω_B, (3) amplification of high-frequency noise less than a factor N_g. These specifications may be reformulated in terms of $\mathcal{H}_\infty$-norm specification using: $W_p(s) = \frac{1}{N_g}\frac{\tau_p s + 1}{\tau_p s + \psi/N_g}, \quad$ with $\tau_p = \frac{1}{N_g \omega_B}$. The Robust Stability and Performance specifications can be coupled into one $\mathcal{H}_\infty$-norm criterion as

$$\sup_\omega \bar{\sigma}(\Psi(s)) \leq 1 \Leftrightarrow ||\Psi(s)||_\infty \leq 1; \quad \text{where } \Psi(s) = \begin{bmatrix} W_d T & 0 \\ 0 & W_p S \end{bmatrix}. \tag{3}$$

ROBUST CONTROLLER DESIGN

Following the problem formulation in section 2, the specifications for controller design are detailed as follows:

1. The design should allow for a worst-case time delay of one minute on the control input (control action) and for ±5% uncertainty in the actuator gains.
2. The final steady-state values of all variables (y_D, x_B) should be within 1 % of their desired values.
3. For distillate product composition y_D, where a change is demanded, the composition should be within ±10% of the desired final value within 40 minutes.
4. The above specifications should be satisfied for both the linear and nonlinear models.

To realize the above specifications, a generalized plant shown is figure 3 is considered.

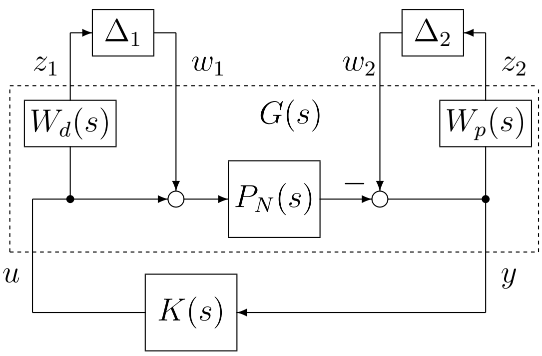

Figure 3: Generalized Plant for μ Synthesis

Here, $w = (w_1, w_2)^T$ are exogenous inputs, $z = (z_1, z_2)^T$ are evaluation signals, u is control input, and y is measured signal. The transfer function which maps w to z is

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} W_dT & W_dKS \\ W_pP_NS & W_pS \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{4}$$

$$z = T_{zw}w \tag{5}$$

where S and T denote sensitivity function and complementary sensitivity function.

From figure 3 and equation (5) the magnitude bound of the uncertainty is formulated as

$$W_d(s) = \mathrm{diag}[W_{d1}(s), W_{d2}(s)], \qquad W_{d1}(s) = W_{d2}(s) = 2\frac{s+0.05}{s+2}. \tag{6}$$

Robust performance weighting function is selected as

$$W_p(s) = \mathrm{diag}[W_{p1}(s), W_{p2}(s)], \quad \text{where} \quad \begin{cases} W_{p1}(s) = 0.5\frac{5s+1}{5s+\xi} \\ W_{p2}(s) = 0.25\frac{10s+1}{10s+\xi} \end{cases} \tag{7}$$

ξ is a substantially small value ($\xi = 10^{-6}$). This robust performance weighting function has been determined to impose strict requirement on the performance of the distillate product composition y_D in order to fulfill the 3rd specification.

A robust performance $\mathcal{H}_\infty$ controller K_μ has been designed based on μ synthesis method [1]. The structured uncertainty is $\mathbf{\Delta}_S = \mathrm{diag}[\Delta_1, \Delta_2]$ where Δ_1 and Δ_2 denote a scalar block and a full block, respectively.

For comparison study, an optimal $\mathcal{H}_\infty$ controller, $K_{H_\infty}(s)$, has also been designed to minimize γ subject to $||T_{zw}(s)||_\infty < \gamma$. Where $T_{zw}(s)$ is given by (5) with the same weighting functions given in (6) and (7). Since the generalized plant does not satisfy the assumption for standard $\mathcal{H}_\infty$ control problem [4], $\mathcal{H}_\infty$ controllers have been designed using LMI approach [5]. The resulted optimum γ is $\gamma^* = 10.8$.

Figure 4 shows step responses when each controller is applied to the nominal plant. It is obvious that $K_\mu(s)$ gives much better tracking performance than K_{H_∞}.

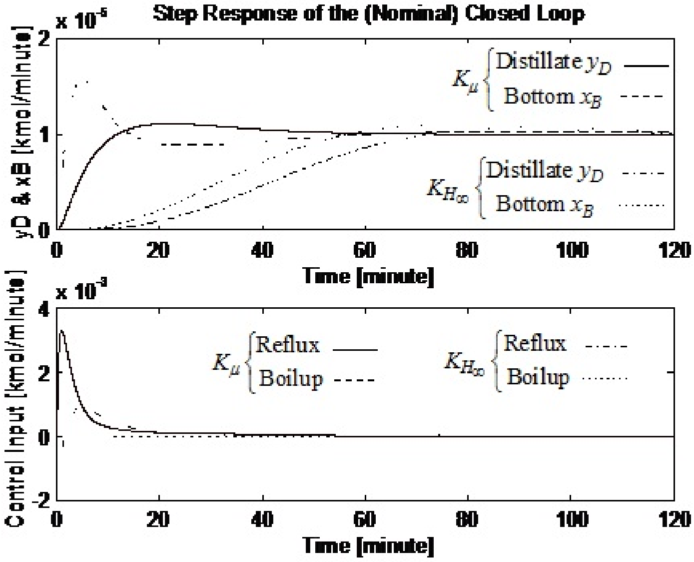

Figure 4: Nominal Step Responses Using $K_\mu(s)$ and K_{H_∞}

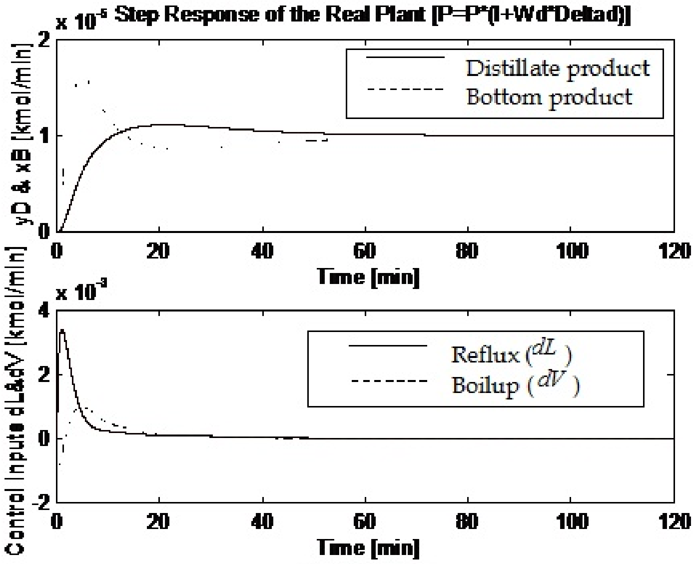

Figure 5: Step Response of Pertubed Real Plant Using $K_\mu(s)$

The robust performance controller $K_\mu(s)$ has been applied to the real plant, that is, the nonlinear dynamic model, with a perturbation of 5% at each control actions dL and dV. Figure 5 shows the step response towards unit step reference dy_D^* and dx_B^*. It can be noted that the closed loop of the perturbed real plant is kept stable, and it responses well with zero steady state error for both bottom product and distillate product, and with short settling time for distillate product y_D. These results exhibit that the given specifications have been achieved.

CONCLUSION AND DISCUSSION

Based on μ synthesis, a robust performance $\mathcal{H}_\infty$ controller has been designed using the nominal plant of order 6^{th}. The controller has been implemented to the real nonlinear plant of distillation column in computer simulation, and it has been proved that the controller satisfies the given design specifications. It has been found that the robust performance $\mathcal{H}_\infty$ controller provides much better performance than optimal $\mathcal{H}_\infty$ controller.

Only perturbation due to the variation in input action is considered in this study. However, external perturbation may also exists in a distillation process. A state-space expression of distillation column dynamics which includes external perturbation may be described as

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ y &= Cx \end{aligned}$$

where w denotes external perturbation. Also, only a certain operating point has been considered in this study. For the case when operating point is often changed, gain scheduling approach may be used. Such these problems are left for further work.

References

[1] A.Packard and J. Doyle,"The Complex Structured Singular Value", Automatica, Vol.29,No.1, pp.71-109, 1993.

[2] S. Skogestad and M. Morari, "LV-Control of a High-Purity Distillation Column", Chemical Engineering Science, Vol.43, No.1, pp.33-48, 1988.

[3] Estiko Rijanto, Niniek F. Puspita, and Kaseno, "Composition Quality Control of Binary Distillation Columns Based on $\mathcal{H}_\infty$ Control Theory", Proc. on TI VI PPI Jepang, Gifu, Japan, August, 1997.

[4] Keith Glover and John C. Doyle, "State-space formulae for all stabilizing controllers that satisfy an $\mathcal{H}_\infty$-norm bound and relations to risk sensitivity", *System & Control Letters* 11, pp.167-172, 1988.

[5] Estiko Rijanto, Antonio Moran, and Minoru Hayase, "Output Feedback $\mathcal{H}_\infty$ Controller for Non-Singular Generalized Plant Using LMI Approach: Controller Structure & Design Procedure", Proc. of The 20th Dynamics System Theory Symposium held by SICE, Sendai, Japan, October 1997.